LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – November 2009

MT 1501 - GRAPHS, DIFF. EQU., MATRICES & FOURIER SERIES

Date & Time: 12/11/2009 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART – A

Answer ALL questions. **(10 x 2 = 20 marks)**

1. Find the domain of the function $\frac{x^2-3x-2}{x^2+x-6}$.

2. If $f: R \to R$ **is defined by** $f(x) = 2x + 5$**, Find** f^{-1} .

3. Write the normal equations of $y = ax + b$.

4. Reduce the equation of the form $y = a + bx + cx^2$ **to linear law.**

5. Form the difference equation of lowest order by eliminating the arbitrary constants a and b from $y = (a + bx)\, 2^x$.

6. Solve $y_{n+2} - y_{n+1} + y_n = 0$.

7. Define row matrix and give an example.

8. Find the characteristic equation of $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

9. Find the Fourier coefficient a_n **for the function** $f(x) = e^x$ **in** $(-\pi, \pi)$.

10. Define periodic function and give an example.

PART – B

Answer any FIVE questions. **(5 x 8 = 40 marks)**

11. The total cost in Rs. Of output x is given by $C = \frac{2}{3}x + \frac{35}{2}$. **Find**

(i) The cost when the output is 4 units.
(ii) The average cost of output of 10 units.
(iii) The marginal cost when the output is 3 units.

12. The cost function for producing x units of a product is $C = x^3 - 12x^2 + 48x + 11$ **(in rupees) and the revenue function is** $R = 83x - 4x^2 - 21$**. Find the output for which the profit is maximum.**

13. Find a straight line fit of the form y = ax + b, by the method of group averages for the following data.

x	0	5	10	15	20	25
y	12	15	17	22	24	30

14. Explain the method of least squares.

15. Solve the difference equation $y_{n+2} - 2y_n \cos\alpha + y_{n-1} = 0$.

16. Find the eigen values and eigen vectors of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$ **.**

17. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ **.**

18. Obtain the Fourier expansion for $f(x) = (\pi - x)^2$ in the interval $(-\pi, \pi)$.

PART – C

Answer any TWO questions. (2 x 20 = 40 marks)

19. (a) From the table given below, find the best values of a and b in the law $y = a e^{bx}$ by the method of least squares.

x	0	5	8	12	20
y	88	87	81	78	74

(b) Suppose that the price and demand for an item are related by $p = 150 - 6x^2$, the demand function where p is the price and x is the number of items demanded (in hundreds). The price and supply are related by $p = 10x^2 + 2$, the supply function where x is the supply of the item (in hundreds). Draw the graph and find the equilibrium demand and equilibrium price.

(12 + 8)

20. Solve the difference equations:

(a) $y_{n+2} - 6y_{n+1} + 8y_n = 4^n$

(b) $y_{n+2} + y_{n+1} - 56y_n = 2^n(n^2 - 3)$. (10 +10)

21. (a) Obtain the cosine series for $x \sin x$ in the interval $(0, \pi)$.

(b) Find a Fourier series expansion for the function $f(x) = \begin{cases} -1 \; for \; -\pi < x < 0 \\ 1 \; for \; 0 \le x \le \pi \end{cases}$ **.**

(10 + 10)

22. Diagonalize the matrix $\begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$ **.**

♬♬♬♬♬♬♬